Roots of Polynomials

Spoken Tutorial Project
http://spoken-tutorial.org
National Mission on Education through ICT
http://sakshat.ac.in

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Learning Objectives

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- **To find extrema and inflection points**

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- **If not, for relevant tutorials, please visit our website www.spoken-tutorial.org**

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- $(a+b)^n = ^nC_0a^n + ^nC_1a^{n-1}b^1 + ^nC_2a^{n-2}b^2 +$ **...** $+ ^nC_ra^{n-r}b^r + ... + ^nC_nb^n$

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- **Reminder,** ${}^nC_1 = \dfrac{n!}{1!(n-1)!}$

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- **When** $\Delta > 0$**, roots are real and unequal**

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- $x_v = \frac{-b}{2a}$ **and** $y_v = ax_v^2 + bx_v + c$

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- **In the complex plane, x axis = real axis, y axis = imaginary axis**

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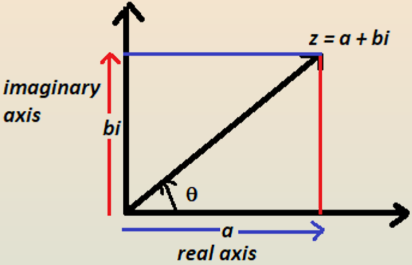

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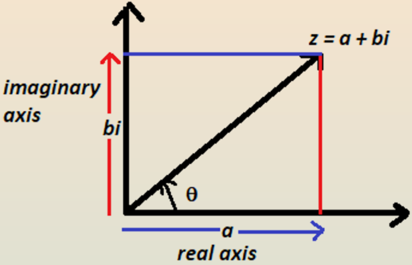

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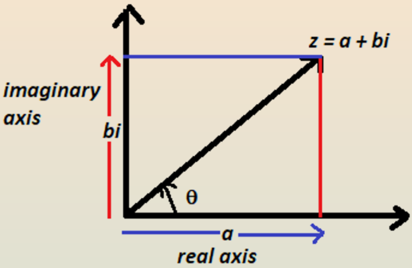

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- Length of vector $z = |z| = r$
- $r = \sqrt{a^2 + b^2}$ (Pythagoras' theorem)

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- **To find co-ordinates of PoI (x, y), we equate 2nd derivative of given function to 0**
- **Solve to get x (x co-ordinate of PoI)**
- **Substitute this x in original function to get y co-ordinate**

Summary

In this tutorial, we have learnt to,

- Plot graphs of polynomial functions using CAS view and input bar
- Find real roots, extrema and inflection point(s)

  Complex roots will be covered in another tutorial

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- $h(x) = (4x + 3)/(x - 1)$
- $i(x) = (3x^2 - 2x - 1)/(2x^2 + 3x - 2)$

About the Spoken Tutorial Project

- Watch the video available at http://spoken-tutorial.org /What_is_a_Spoken_Tutorial
- It summarizes the Spoken Tutorial project
- If you do not have good bandwidth, you can download and watch it

Spoken Tutorial Workshops

The Spoken Tutorial Project Team

- Conducts workshops using spoken tutorials
- Gives certificates to those who pass an online test
- For more details, please write to contact@spoken-tutorial.org

Forum for specific questions

- Do you have questions in THIS Spoken Tutorial?
- Please visit http://forums.spoken-tutorial.org
- Choose the minute and second where you have the question
- Explain your question briefly
- Someone from our team will answer

Acknowledgements

- Spoken Tutorial Project is a part of the Talk to a Teacher project
- It is supported by the National Mission on Education through ICT, MHRD, Government of India
- More information on this Mission is available at http://spoken-tutorial.org /NMEICT-Intro