Additional Reading Material

Flow in a Convergent Divergent Nozzle

Nozzle Flows

The isentropic flow of a calorically perfect gas through a nozzle is governed by the relation:

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{(\gamma+1)/(\gamma-1)}$$

The Mach number at any given point in a nozzle is a function of ratio of local throat area to sonic throat area. There exists two values of Mach number for given area ratio which are both subsonic and supersonic values. The pressure at the inlet and exit of nozzle dictates only one value of Mach number in a given case.

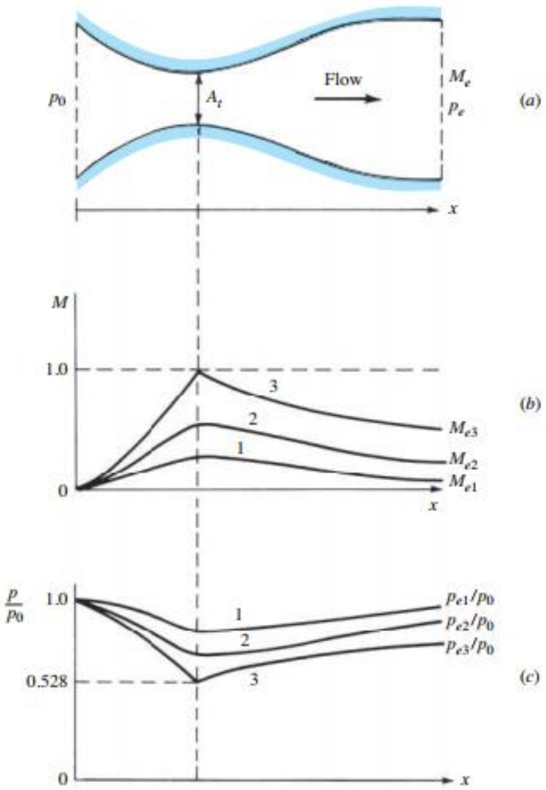

Fig 1: Isentropic subsonic flow in a nozzle (Anderson, 2011)

At a certain exit pressure (when $p_e = p_{e,6}$), there will be supersonic isentropic flow in a nozzle. If the exit pressure is slightly less than p_0 (when $p_e = p_{e,1}$) , there won't be sonic condition in the throat and Mach number decreases downstream the throat. As we decrease the exit pressure such that $p_e = p_{e,3}$, there will be sonic condition in the throat but the flow downstream the throat still remain subsonic as shown in Fig 1. As we further decrease the exit pressure, the mass flow rate will remain constant and flow will be sonic in the throat, this condition is known as choked condition.

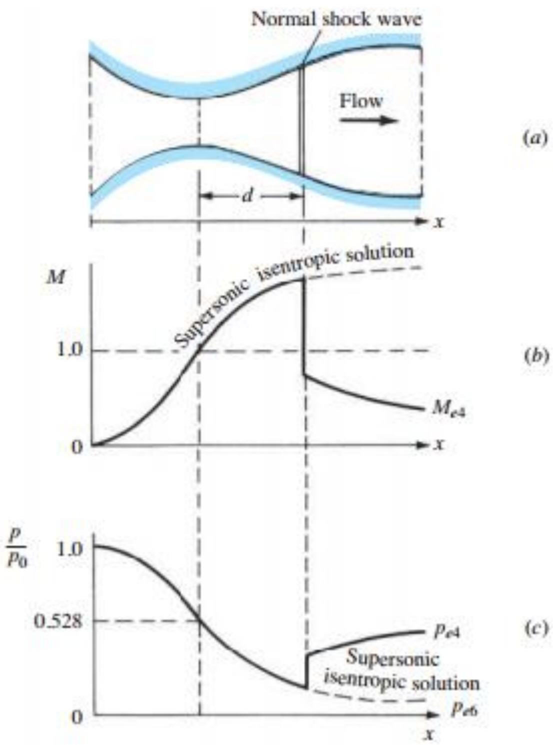

Fig 2: Supersonic nozzle with normal shock wave inside the nozzle (Anderson, 2011)

As the exit pressure is reduced below $p_{e,6}$, a region of supersonic flow appears downstream of the throat. However, the exit pressure is too high to allow an isentropic supersonic flow throughout the entire divergent section. Instead, for p_e less than $p_{e,3}$ but substantially higher than the fully isentropic value $p_{e,6}$, a normal shock wave is formed downstream of the throat as shown in Fig 2.

Calculation

In the present tutorial, the supersonic inlet pressure is 10^4 Pa and area ratio of exit and throat is 2.5. From Appendix A (Anderson, 2011), the ratio of stagnation inlet pressure and exit pressure is 15.81 for supersonic Mach number.

$$\frac{P_e}{P_0} = \frac{1}{15.81}$$

Therefore, $$P_e = 632\ Pa$$

Also, From Appendix A (Anderson, 2011), the ratio of stagnation inlet pressure and exit pressure is 1.048 for subsonic Mach number.

$$\frac{P_e}{P_0} = \frac{1}{1.048}$$

Therefore, $$P_e = 9541.9\ Pa$$

The isentropic supersonic and subsonic exit pressure is calculated as 632 Pa and 9541.9 Pa respectively for the area ratio of exit and throat as 2.5. So, **7500 Pa** is taken as exit pressure in the tutorial as it is one of the exit pressure values which allow normal shock wave to form in the divergent section of the nozzle. Note: This is a static pressure.

Regarding the boundary condition, the stagnation pressure of 10000 Pa is applied as a pressure inlet. This boundary condition is applied when the velocity at the inlet is unknown and used in supersonic compressible and buoyancy driven flow. The velocity is calculated by using static pressure from previous iteration using following formula:

$$U = M\sqrt{\gamma RT} \qquad\qquad M = \sqrt{\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

And, static pressure is updated using this velocity and the iteration continues.

References

1) Anderson, J. D. (2011). *Fundamentals of Aerodynamics.* McGraw hill.