Linear Programming with two-index variables in Scilab

Spoken Tutorial Project

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In this tutorial, we will learn how to:

In this tutorial, we will learn how to:

- Solve a Linear Programming problem, having variables with two indices in Scilab.

Objective

In this tutorial, we will learn how to:

- **Solve a Linear Programming problem, having variables with two indices in Scilab.**
- **Use Karmarkar function.**

In this tutorial, we will learn how to:

- Solve a Linear Programming problem, having variables with two indices in Scilab.
- Use Karmarkar function.
- Use Transportation Problem as an example of Linear Programming.

- You should have gone through the spoken tutorial on Optimization Using Karmarkar Function

Prerequisites

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- **This tutorial is available on http://spoken-tutorial.org.**

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- **Be familiar with Linear Programming.**
- **Have Scilab installed on your system.**

Terminology

We will refer to:

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- Linear Programming as LP.

Terminology

We will refer to:

- **Linear Programming as LP.**
- **Transportation Problem as TP.**

What is a two-index Variable?

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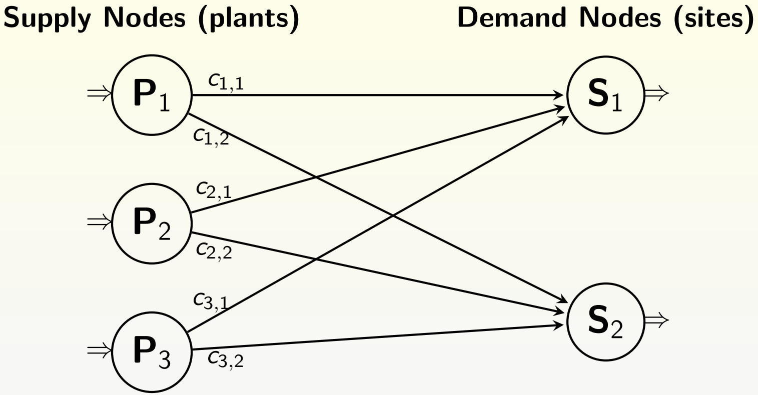
- **For example** $x_{1,2}$
- **An example of LP with two-index variables is the TP.**

What is Transportation Problem?



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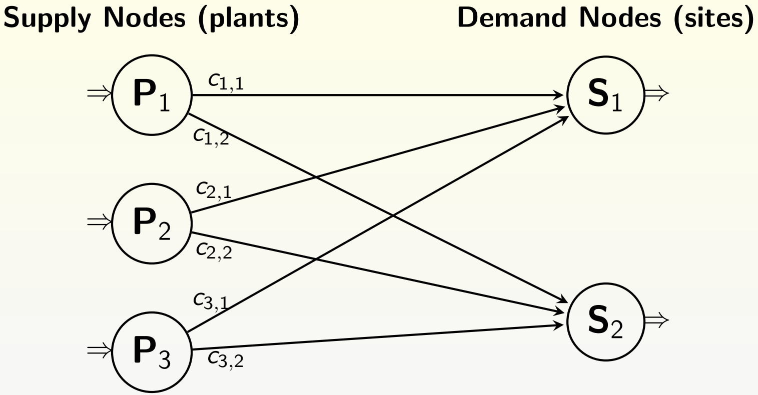
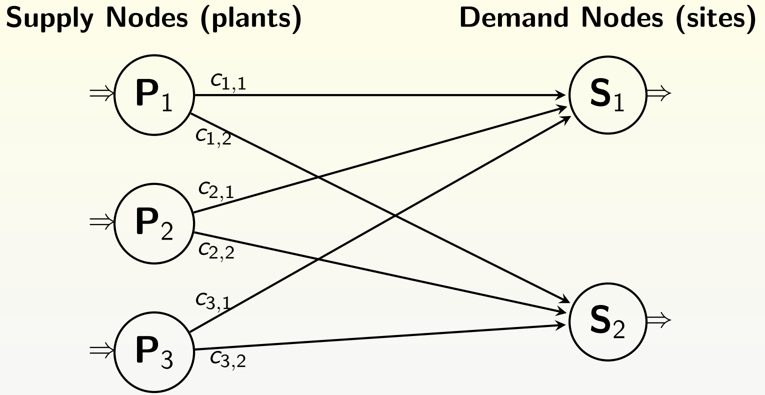


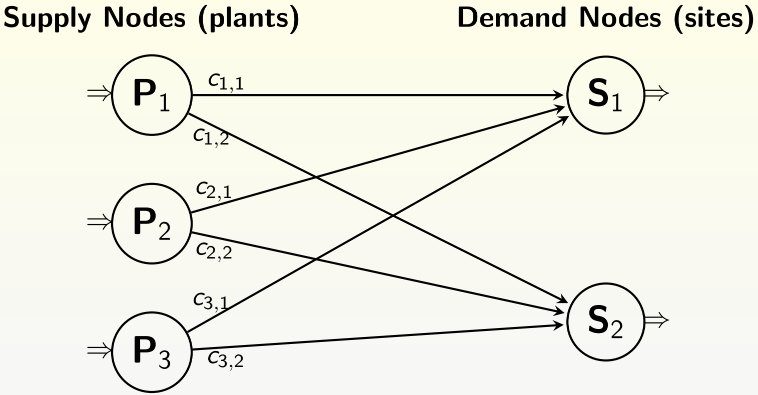
- AIM: To supply goods from plants to sites, with total minimum transportation cost.

What is Transportation Problem?



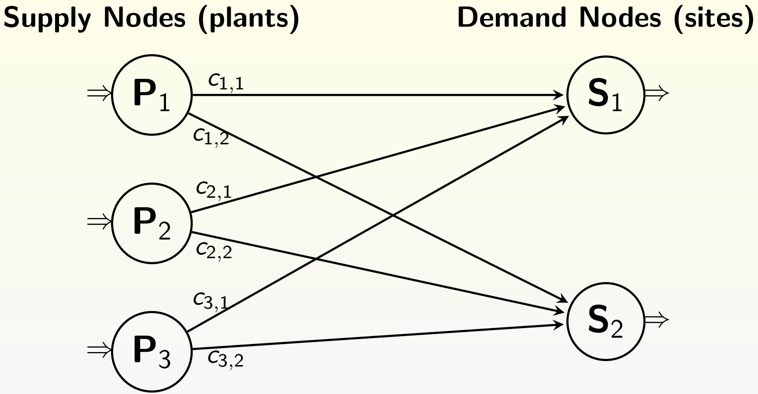
- Supply nodes cannot transport more than their capacity.

What is Transportation Problem?



- Supply nodes cannot transport more than their capacity.
- All the demands at the sites must be met.

What is Transportation Problem?



- Supply capacities, demands and transportation costs are known.

- **A cement company transports cement from plants 1, 2 and 3 to construction sites 1 and 2.**

Transportation Problem Example

- **A cement company transports cement from plants 1, 2 and 3 to construction sites 1 and 2.**
- **The available supply at each plant is: plant-1: 45 tons, plant-2: 60 tons and plant-3: 35 tons.**

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- **The available supply at each plant is: plant-1: 45 tons, plant-2: 60 tons and plant-3: 35 tons.**
- **The demands of sites are: site-1: 50 tons and site-2: 60 tons.**

Transportation Problem Example

- The cost of transporting 1 ton of cement from each plant to each site, is given in the table:

	Transportation Cost		
Plants	**site-1**	**site-2**	**Available Supply**
plant-1	3	2	45
plant-2	1	5	60
plant-3	5	4	35
Demand	50	60	

Transportation Problem Example

- The cost of transporting 1 ton of cement from each plant to each site, is given in the table:

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Demand	50	60	

- We will formulate it as LP problem and find its optimal solution.

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- **Two-index decision variable: $x_{i,j}$ is the number of tons transported from plant i to site j.**

Formulation

	Transportation Cost		
Plants	site-1	site-2	Available Supply
plant-1	3	2	45
plant-2	1	5	60
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- Objective Function

  Minimize

  $3x_{1,1} + 2x_{1,2} + x_{2,1} + 5x_{2,2} + 5x_{3,1} + 4x_{3,2}$

Formulation

	Transportation Cost		
Plants	**site-1**	**site-2**	**Available Supply**
plant-1	**3**	**2**	**45**
plant-2	**1**	**5**	**60**
plant-3	**5**	**4**	**35**
Demand	**50**	**60**	

- Constraints:

  $x_{1,1} + x_{1,2} \leq 45$

  $x_{2,1} + x_{2,2} \leq 60$

  $x_{3,1} + x_{3,2} \leq 35$

	Transportation Cost		
Plants	site-1	site-2	Available Supply
plant-1	3	2	45
plant-2	1	5	60
plant-3	5	4	35
Demand	50	60	

- Constraints:

$$x_{1,1} + x_{2,1} + x_{3,1} \geq 50 \Leftrightarrow -x_{1,1} - x_{2,1} - x_{3,1} \leq -50$$
$$x_{1,2} + x_{2,2} + x_{3,2} \geq 60 \Leftrightarrow -x_{1,2} - x_{2,2} - x_{3,2} \leq -60$$

Formulation

	Transportation Cost		
Plants	site-1	site-2	Available Supply
plant-1	3	2	45
plant-2	1	5	60
plant-3	5	4	35
Demand	50	60	

- $x_{i,j} \geq 0 \ (i = 1, 2, 3; \ j = 1, 2)$

- **Karmarkar function is LP solver in Scilab.**

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- **Karmarkar function solves the LP problem in the following form.**

$$\begin{aligned} Minimize\ \ c^T x \\ subject\ \ to: A_{eq}x = b_{eq}, \\ Ax \leq b, \\ lb \leq x \leq ub, \end{aligned}$$

where

- c^T, A_{eq}, A, b_{eq}, b, lb **and** ub **are known parameters.**

$$
\begin{aligned}
Minimize\ c^T x \\
subject\ to: A_{eq}x = b_{eq}, \\
Ax \leq b, \\
lb \leq x \leq ub,
\end{aligned}
$$

where

- c^T, A_{eq}, A, b_{eq}, b, lb **and** ub **are known parameters.**
- x **is the vector of decision variables.**

Input Parameters

$$Minimize\ 3x_{1,1} + 2x_{1,2} + x_{2,1} + 5x_{2,2} + 5x_{3,1} + 4x_{3,2}$$

$$\begin{aligned} subject\ \ to: x_{1,1} + x_{1,2} &\leq 45, \\ x_{2,1} + x_{2,2} &\leq 60, \\ x_{3,1} + x_{3,2} &\leq 35, \\ -x_{1,1} - x_{2,1} - x_{3,1} &\leq -50, \\ -x_{1,2} - x_{2,2} - x_{3,2} &\leq -60, \end{aligned}$$

$$x_{i,j} \geq 0\ (i = 1, 2, 3;\ j = 1, 2).$$

- The coefficient vector of objective function.

$$c^T = \begin{bmatrix} 3 & 2 & 1 & 5 & 5 & 4 \end{bmatrix}$$

Input Parameters

- **The coefficient matrix of constraints.**

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ -1 & 0 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 \end{bmatrix}$$

Input Parameters

$$\textit{Minimize } 3x_{1,1} + 2x_{1,2} + x_{2,1} + 5x_{2,2} + 5x_{3,1} + 4x_{3,2}$$

$$\begin{aligned} \textit{subject to}: x_{1,1} + x_{1,2} &\leq 45, \\ x_{2,1} + x_{2,2} &\leq 60, \\ x_{3,1} + x_{3,2} &\leq 35, \\ -x_{1,1} - x_{2,1} - x_{3,1} &\leq -50, \\ -x_{1,2} - x_{2,2} - x_{3,2} &\leq -60, \\ x_{i,j} \geq 0 \ (i = 1, 2, 3;\ j = 1, 2). \end{aligned}$$

Input Parameters

$$\begin{aligned} \textit{Minimize } & 3x_{1,1} + 2x_{1,2} + x_{2,1} + 5x_{2,2} + 5x_{3,1} + 4x_{3,2} \\ \textit{subject to : } & x_{1,1} + x_{1,2} \le 45, \\ & x_{2,1} + x_{2,2} \le 60, \\ & x_{3,1} + x_{3,2} \le 35, \\ & -x_{1,1} - x_{2,1} - x_{3,1} \le -50, \\ & -x_{1,2} - x_{2,2} - x_{3,2} \le -60, \\ & x_{i,j} \ge 0 \ (i = 1, 2, 3; \ j = 1, 2). \end{aligned}$$

- The vector of right hand side of constraints.

$$b^T = \begin{bmatrix} 45 & 60 & 35 & -50 & -60 \end{bmatrix}$$

Input Parameters

$$Minimize\ 3x_{1,1} + 2x_{1,2} + x_{2,1} + 5x_{2,2} + 5x_{3,1} + 4x_{3,2}$$

$$\begin{aligned} subject\ to: x_{1,1} + x_{1,2} &\leq 45, \\ x_{2,1} + x_{2,2} &\leq 60, \\ x_{3,1} + x_{3,2} &\leq 35, \\ -x_{1,1} - x_{2,1} - x_{3,1} &\leq -50, \\ -x_{1,2} - x_{2,2} - x_{3,2} &\leq -60, \\ x_{i,j} \geq 0\ (i = 1, 2, 3;\ j &= 1, 2). \end{aligned}$$

- Lower bound for each variable.

$$lb = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Input Parameters

$$Minimize\ 3x_{1,1} + 2x_{1,2} + x_{2,1} + 5x_{2,2} + 5x_{3,1} + 4x_{3,2}$$

$$\begin{aligned} subject\ \ to: x_{1,1} + x_{1,2} &\leq 45, \\ x_{2,1} + x_{2,2} &\leq 60, \\ x_{3,1} + x_{3,2} &\leq 35, \\ -x_{1,1} - x_{2,1} - x_{3,1} &\leq -50, \\ -x_{1,2} - x_{2,2} - x_{3,2} &\leq -60, \\ x_{i,j} \geq 0\ (i = 1, 2, 3;\ j &= 1, 2). \end{aligned}$$

- In our example, we do not require A_{eq}, b_{eq} and ub.

Steps to download the required code

- **Pause this tutorial.**

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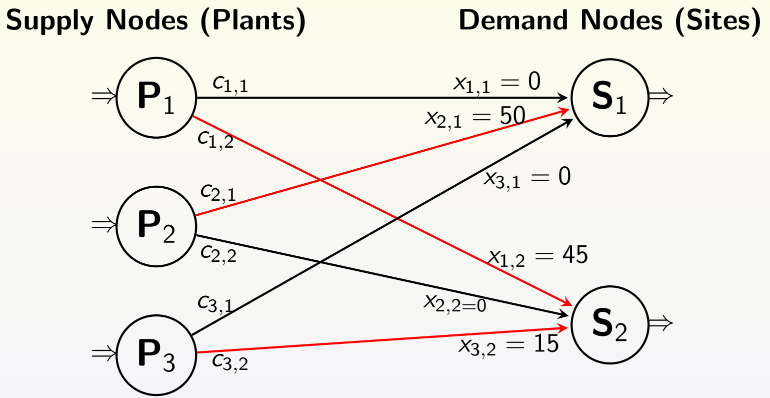
- **Locate this file and open it in the Scilab console.**

Result


Supply Nodes (Plants)
Demand Nodes (Sites)
P1
P2
P3
S1
S2
c1,1
c1,2
c2,1
c2,2
c3,1
c3,2
x1,1 = 0
x2,1 = 50
x3,1 = 0
x1,2 = 45
x2,2=0
x3,2 = 15

Assignment

	Transportation Cost		
Plants	site-1	site-2	Available Supply
plant-1	5	2	90
plant-2	4	3	160
Demand	100	60	

- Hint: Formulate this problem as explained in the previous example.

	Transportation Cost		
Plants	site-1	site-2	Available Supply
plant-1	5	2	90
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Demand	100	60	

- Hint: Formulate this problem as explained in the previous example.
- The optimal value should be 520.

	Transportation Cost		
Plants	site-1	site-2	Available Supply
plant-1	5	2	90
plant-2	4	3	160
Demand	100	60	

- Hint: Formulate this problem as explained in the previous example.
- The optimal value should be 520.
- The optimal solution should have $x_{1,2}$=60, $x_{2,1}$=100 and all others have zero.

Summary

In this tutorial, we have learnt

- About variables with two indices.
- Example of LP with two-index variables and its input parameters.
- Applying Karmarkar function to solve Linear Transportation problem in Scilab.

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- This project promotes the use of free and open source software tools.
- For more details, please visit: http://fossee.in/

- Watch the video available at http://spoken-tutorial.org /What_is_a_Spoken_Tutorial.
- It summarises the Spoken Tutorial project.
- If you do not have good bandwidth, you can download and watch it.

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