Curve Fitting

There are different types of algebraic equations, of which we will look at linear and quadratic for this PhET. A **polynomial** is an algebraic expression consisting of terms. It is of the form, $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$.

Software like **GeoGebra** (refer to tutorials in spoken-tutorial.org) are very useful to explore the effects of all coefficients on graphs.

Quadratic equations are of the general form $\mathbf{y = ax^2 + bx + c}$. Here, the exponent of 2 is the highest and is the degree of the equation. Such equations are graphed as **parabolas**. Their vertex is given by (x_v, y_v), where $x_v = –b/2a$ and $y_v = ax_v^2+bx_v+c$.

- They open upwards if $a > 0$ and open downwards if $a < 0$. The graph is a line when $a = 0$.
- If $b=c=0$, $y=ax^2$, then as a increases, the parabola becomes narrower or "slimmer" and its vertex is at the origin (0,0).
- If $c=0$, $y=ax^2+bx$, parabolas (not vertex) all pass through the origin (0,0).
    - For a given value of $a > 0$, increasing b moves the vertex to the left.
        - If $b < 0$, vertex moves upwards
        - If $b > 0$, vertex moves downwards
    - For a given value of $a < 0$, increasing b moves the vertex to the right.
        - If $b < 0$, vertex moves downwards
        - If $b > 0$, vertex moves upwards
- c is the point of intersection between the parabola and the y-axis. Increasing c makes the vertex move upwards

When a=0, **y=bx+c** is a **linear** equation. Linear equations are of the form $y = mx + b$ where m cannot be 0. The degree of this equation is 1 as the highest exponent of x in this equation is 1. When graphed, linear equations describe lines. In $y = mx + b$, m is called the **slope** or gradient and b is the **y intercept** where the line intersects the y-axis.

Equations of the form **y = c**, where c is any constant, also describe lines. However, such lines are parallel to the x-axis and pass through (0,c). Their slope is 0.

On the other hand, the y-axis, described by equation $x = 0$, and lines of the form **x = c**, which are parallel to the y-axis, have slope of infinity (∞).

Constant (non-zero), linear, quadratic, cubic and quartic polynomials are of degree 0, 1, 2, 3 and 4, respectively.

Graphs of polynomial functions can be predicted based on their degree, roots and the signs of their first and second derivatives.

For $f(x) = (x-a)(x-b)(x-c)(x+d)$, its roots are a, b, c and –d. **Roots** are the solutions to the equations of their functions when the functions are equated to 0.

In the case of f(x), all roots have multiplicity of 1. As 1 is odd, the graph for f(x) cuts the x-axis at (a,0), (b,0), (c,0) and (-d,0). However, for $g(x)=(x-p)^2 (x-q)^3$, there are two repeated roots, x = p with multiplicity 2 and x = q with multiplicity 3. As 2 is even, the graph of g(x) touches the x axis at (p,0). As 3 is odd, the graph of g(x) cuts through the x axis at (q,0).

Graphically, when the roots are real and unequal, the graph is a parabola that intersects the x axis at two points. When the roots are real but equal, the graph is a parabola that touches the x axis at one point. However, when the roots are complex, the graph is a parabola that never intersects the x axis. Its two complex (or imaginary) roots are of the form $a + bi$.

Also helpful in graphing polynomials is knowledge of its **inflection points** (where the curve changes sign or concavity) and **extrema** (**maxima** or **minima**). There are many resources (textbooks, You Tube videos, teaching materials from colleges and universities) that will help you explore polynomials and their graphs and solutions. Also check out **Pascal's triangle** which is an easy way to find the coefficients arising from binomial expansion.

A polynomial of degree 3 is called a **cubic polynomial**. It is of the general form $\mathbf{y = ax^3+bx^2+cx+d}$. Such a function has a maximum of 3 roots. If all 3 are real and distinct, the graph intersects the x-axis 3 times. A cubic polynomial has one inflection point, can have 0 or 2 extrema, and 3 fundamental shapes.

A polynomial of degree 4 is called a **quartic polynomial**. It is of the general form $y = ax^4+bx^3+cx^2+dx+e$. Such a function has a maximum of 4 roots. It may have 1, 2 or 3 extrema; 0, 1 or 2 inflection points; and 7 fundamental shapes.

The reduced chi-squared statistic $\boldsymbol{\chi_r^2}$ is used to test "**goodness of fit**". Its square root is called standard error of regression. It is the chi-squared value per degree of freedom. This is useful when the audience is not familiar with the data and does not know how many degrees of freedom apply in your case. If the value of $\boldsymbol{\chi_r^2}$ is close to 1, as long as there are enough data points, there is a good match between observations and estimates. If $\boldsymbol{\chi_r^2} < 1$, the fit is "too good" and the variance has been overestimated. If $\boldsymbol{\chi_r^2} > 1$, the fit has not captured the data. The higher the value of $\boldsymbol{\chi_r^2}$, the poorer is the fit.